Hall Ticket Number:

Code No. : 14109 LAA

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) IV-Semester Main Examinations, May-2018

Linear Algebra and its Applications

(Open Elective-III)

Time: **3** hours Max. Marks: **70**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

1. Determine whether the subset $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| x_1 x_2 x_3 = 0 \right\}$ is a subspace of R^3.

2. Explain why the Set $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \right\}$ is not a basis for the vector space $V = M_{2 \times 2}$.

3. Find the coordinates of the vector $v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ relative to the ordered basis $B = \left\{ \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$.

4. Determine whether the function $T : R^2 \to R^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + y \\ x - y + 2 \end{bmatrix}$ is a linear transformation.

5. Find the basis for the null space of the linear transformation $T : R^2 \to R^2$, defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} -x + y \\ x - y \end{bmatrix}$.

6. A linear operator $T : R^2 \to R^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} -x \\ y \end{bmatrix}$, find the matrix representation for T relative to the standard basis for R^n.

7. Find a scalar c, so that $\begin{bmatrix} c \\ 3 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

8. Prove that Pythagorean theorem to R^n.

9. Let $V = R^2$, with inner product defined by $\langle u, v \rangle = u_1 v_1 + 3 u_2 v_2$. Verify that the Cauchy-Schwartz Inequality is upheld.

10. Let $V = P_2$, with inner product defined by $\langle p, q \rangle = \int_{-1}^{1} p(x) q(x) dx$. Find the length of each vector in S. where $S = \{1, x, 1/2\,(3x^2 - 1)\}$.

Part-B (5 × 10 = 50 Marks)

11. a) Find the coordinates of the vector **v** relative to the two ordered bases B_1 and B_2 [5]
$B_1 = \{x^2 - x + 1, x^2 + x + 1, 2x^2\}$
$B_2 = \{2x^2 - x + 1, -x^2 + x + 2, x + 3\}$
$\mathbf{v} = p(x) = x^2 + x + 3$

b) Find a basis for the **span**(S) as a subspace of R^3 where [5]

$$S = \left\{ \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix} \right\}$$

12 a) Let $S = \left\{ \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ [5]

i) Find span S.
ii) Is S linearly independent?

b) Explain sub-space of a vector space with an example. [5]

13 a) A linear operator $T : P_2 \to P_2$, define as [5]

$$T(p(x)) = p'(x) + p(x)$$
$$B = \{1 - x - x^2, 1, 1 + x^2\}$$
$$B' = -1 + x, -1 + x + x^2, x\}$$
$$\mathbf{v} = 1 - x$$

i) Find the matrix representation for T relative to the standard basis for R^n.
ii) Find **T(v)**, using a direct computation and using the matrix representation.

b) Verify the dimensionality theorem for the linear operator $T : R^3 \to R^3$ and $B = \{v_1, v_2, v_3\}$ a basis for R^3. Suppose that [5]

$$T(v_1) = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \quad T(v_2) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad T(v_3) = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

14 a) Let $T : R^3 \to R^3$ be a linear transformation, and let **B** be the standard basis for R^3. [5]

$$T(e_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad T(e_2) = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad T(e_3) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{Find } T(\mathbf{v}), \text{ where } \mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$$

b) If $T : P_2 \to P_2$ is a linear operator and [5]
$T(1) = 1 + x$; $T(x) = 2 + x^2$; $T(x^2) = x - 3x^2$ then find T $(-3 + x - x^2)$.

15. Let B be the basis for R^3 given by $B = \{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ [10]

Apply the Gram-Schmidt process to B to find an orthogonal basis for R^3.

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